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Adaptive Observation Covariance for EKF–SLAM in Indoor Environments using Laser Data

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Abstract— **In this paper we describe an approach to concurrently localize a robot and to build a feature based map using laser sensor. Stochastic simultaneous localization and mapping (SLAM) is performed by storing the robot pose and map landmarks in a single state vector, and estimating this state vector via a recursive process of prediction and updating. In our case, these estimates are updated using an extended Kalman filter (EKF). The main novelty of this proposal is the development and test of an adaptive measurement covariance matrix that permits to include close and distant features in the updating stage of the EKF–SLAM algorithm, providing more precision to closer detected features.**

I. Introduction

Simultaneous Localization and Mapping (SLAM) is an essential ability for autonomous mobile robots exploring unknown environments. The problematic of SLAM lies in the fact that an accurate estimation of the robot pose is required to obtain a good map of the environment, and to reduce the unbounded growing odometry errors requires to associate sensor measurements with a precise map [10]. The efficiency and robustness of the SLAM process is determined by the chosen map representation [8]. Typical choices for this representation include occupancy grids [9], topological maps [5] and feature maps [6].

In this paper, a feature based approach is employed to solve the SLAM problem. Feature maps are a viable representation for long-term convergent SLAM in medium-scale environments [1]. It allows the use of multiple models to describe the measurement process for different parts of the environment and it avoids the data smearing effect [10]. Feature map SLAM implies to add observed landmarks to the map using the robot pose as a reference, while existing map landmarks are employed to estimate the robot pose. Then, the uncertainty of sensor measurements provokes uncertain estimates of all parameters, i.e. the robot pose and the map landmark locations, and these uncertainties are correlated [1]. Consistent stochastic estimation requires that correlations between parameters are explicitly maintained. In our case, a basic stochastic SLAM algorithm is used. This algorithm stores all parameters in a state vector and updates these estimates using an extended Kalman filter (EKF).

In the standard EKF–based approach to SLAM, the robot pose and landmark locations are represented by a stochastic state vector. The stochastic SLAM is performed by estimating the state vector via a recursive process of prediction, data association and updating. The prediction stage provides an estimation of the robot pose based on odometry. Then, landmarks are extracted from the environment and associated to previously stored ones. The updating stage employs the data association results to determine the posterior estimated robot pose. In this last stage, each landmark is typically characterized by its location relative to the robot and by an observation covariance. This observation covariance models the error of the data acquisition system. The covariance matrix values are experimentally estimated and are usually constant for every landmark location. In our case, features are extracted from laser data and, therefore, observation covariance values are low. However, in order to avoid the acquisition of false landmarks, several authors propose to do not take into account landmarks which are more distant from the robot that a fixed threshold [1]. This reduction of the set of detected landmarks can be specially negative for the EKF–SLAM approach when the environment does not provide a dense map of landmarks. In these case, the updating rate is reduced and the robot pose uncertainty can be excessively increased. In this paper, we propose an observation covariance whose values depend on the distance between robot and detected landmarks. Thus, distant landmarks can be used to update the robot pose, although the observation covariance associated to these landmarks will be higher than the associated to closer landmarks.

The rest of the paper is organized as follows: The landmark acquisition and data association using laser data are presented in Section 2. This section also describes how is obtained the adaptive observation covariance and its application in the landmark uncertainty. Section 3 presents experimental results and, finally, Section 4 summarizes conclusions and future work.

II. Laser Data Landmark Acquisition and Association using an adaptative observation covariance

In the standard EKF–based approach to SLAM, the robot pose and landmark locations at time step k are represented by a stochastic state vector $\mathbf{x}_a^k$ with estimated mean $\hat{\mathbf{x}}_a^k$ and estimated error covariance $\mathbf{P}_a^k$. This state vector contains an

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estimation of the robot pose, $\hat{\mathbf{x}}_v^k$, and the estimated environment landmarks positions, $\hat{\mathbf{x}}_m^k$, all with respect to a base reference W. This concatenation is necessary as consistent SLAM relies on the maintenance of correlations $\mathbf{P}_{vm}^k$ between robot and map [1]. In this work, we use the robot pose before the first motion (at step k=0) as the base reference ($W = \mathbf{x}_v^0$), because it improves the consistency of the process [3]. Thus, the map can be initialized with zero covariance for the robot pose, $\hat{\mathbf{x}}_a^0 = 0$, $\mathbf{P}_a^0 = 0$. For convenience, the k notation can be dropped in this section as the sequence of operations is apparent from its context. Then, the mean $\hat{\mathbf{x}}_a$ and covariance $\mathbf{P}_a$ of the state vector can be defined as

$$\hat{\mathbf{x}}_a = \begin{bmatrix} \hat{\mathbf{x}}_v \\ \hat{\mathbf{x}}_m \end{bmatrix} \qquad \mathbf{P}_a = \begin{bmatrix} \mathbf{P}_{vv} & \mathbf{P}_{vm} \\ \mathbf{P}_{vm}^T & \mathbf{P}_{mm} \end{bmatrix} \tag{1}$$

When the robot pose and map landmarks are stored in a single state vector, stochastic SLAM [1] is performed by estimating the state parameters via a recursive process of prediction, landmark observation and association and updating. The prediction stage deals with robot motion based on incremental dead reckoning estimates, and increases the uncertainty of the robot pose estimate. Then, new landmarks are acquired from the environment. These landmarks are associated to the previously stored ones. The update stage uses this data association to improve the overall state estimate. Finally, if the data association process determines that a new landmark not stored in the map have been observed, it will be added to the state vector through an initialization process called state augmentation. This state augmentation stage must be carefully performed in order to avoid the inclusion of false landmarks in the state vector.

Laser sensor typically provides dense information of the environment in a single scan which presents high angular precision. Typical feature based approaches to SLAM which use laser data perform features acquisition and data association tasks in two consecutive steps. Firstly, a segmentation step, where the laser range readings are processed to obtain simple geometric features such as line segments, curve segments or corners (landmark acquisition). Then, during the map building process, a second stage looks for matches between the features obtained from different scans, based on a probabilistic model of the sensor and the robot motion (data association). On the contrary, other approaches employ raw laser data as map feature and solve the data association by computing the robot pose that maximizes scan-to-scan [4] or scan-to-map [11] correlations. In these approaches, the density and precision of the laser data is fundamental to achieve a robust scan matching process. In this work, we differentiate feature acquisition and data association tasks. Thus, next sections describe the landmark acquisition and data association stages of our proposed EKF–SLAM approach. Section II-C deals with the adaptive observation covariance calculation and its application in the EKF updating stage. Finally, the state augmentation stage is briefly described.

A. *Corner acquisition from laser data*

In this work, we employ the curvature information to characterise the local planar scan provided by the laser sensor. The curvature index at each range reading of the laser scan is adaptively filtered according to the distance between possible corners in the whole laser scan. This method permits to remove noise, but scan features are nevertheless detected despite their natural scale. For each range reading i of a laser scan, the proposed method for adaptive curvature estimation in laser scan data consists of the following steps:

1) Calculation of $K_f(i)$ and $K_b(i)$, the maximum length of laser scan presenting no discontinuities on the right and left sides of the range reading i respectively. $K_f(i)$ is calculated by comparing the Euclidean distance from i to its $K_f(i)$-th neighbour ($d(i, i+K_f(i))$) to the real length of the laser scan between both range readings($l(i, i + K_f(i))$). These distances tend to be the same in absence of corners, even if laser scans are noisy. Otherwise, the Euclidean distance is quite shorter than the real length. Thus, $K_f(i)$ is the largest value that satisfies
$$d(i, i + K_f(i)) > l(i, i + K_f(i)) - U_k \tag{2}$$
where U_k is a constant value that depends on the noise level tolerated by the detector. $K_b(i)$ is also set according to Eq. (2), but using $i - K_b(i)$ instead of $i+K_f(i)$. In our case, it has been experimentally proven that U_k equal to 1.0 works correctly.
2) Calculation of the local vectors $\overrightarrow{f_i}$ and $\overrightarrow{b_i}$ associated to each range reading i. These vectors present the variation in the x and y axis between range readings i and $i + K_f(i)$ and between i and $i - K_b(i)$. If (x_i, y_i) are the coordinates of the range reading i, the local vectors associated to i are defined as
$$\begin{aligned} \overrightarrow{f_i} &= (x_{i+K_f(i)} - x_i, y_{i+K_f(i)} - y_i) = (f_{x_i}, f_{y_i}) \\ \overrightarrow{b_i} &= (x_{i-K_b(i)} - x_i, y_{i-K_b(i)} - y_i) = (b_{x_i}, b_{y_i}) \end{aligned} \tag{3}$$
3) Calculation of the angle associated to i. According to previous works [7], the angle at range reading i can be estimated as follows
$$|K_\theta(i)| = \frac{1}{2} \cdot \left(1 + \frac{\overrightarrow{f_i} \cdot \overrightarrow{b_i}}{|\overrightarrow{f_i}| \cdot |\overrightarrow{b_i}|} \right) \tag{4}$$
4) Detection of corners over $|K_\theta(i)|$. The obtained curvature index represents the curvature associated to each range reading in an absolute manner. Corners are those range readings which satisfy the following conditions: i) they are local peaks of the curvature function and ii) their $|K_\theta(i)|$ values are over the minimum angle required to be considered a corner instead of a spurious peak due to remaining noise (θ_{min}).

B. *Observed and stored corner association*

Once corners have been extracted from the laser data, they must be associated to previously stored ones. Correct correspondence of observed and stored corners is essential for consistent map building and robot pose estimation. Thus, successful data association involves the detection and rejection of spurious measurement, because a single failure may invalidate all the SLAM process. When corners are considered as landmarks, the map is represented by a set of practically identical features, distinguishable only by their positions. Because of this representation, correspondences established by the data association stage are constrained by statistical geometric information.

In this work, we perform the association validation in observation space using the innovation sequence and its predicted covariance. The innovation sequence ν_{ij} relates observed measurement $\mathbf{z}$ to the predicted observation $h(\hat{\mathbf{x}}_j)$ for target $\mathbf{x}_j$ by the difference $\nu_{ij} = \mathbf{z}_i - h(\hat{\mathbf{x}}_j)$ [1]. Given an observation innovation ν_{ij} with covariance $\mathbf{S}_{ij}$, the normalised innovation squared (NIS) is defined as [2]

$$NIS \equiv \nu_{ij}^T \mathbf{S}_{ij}^{-1} \nu_{ij} \tag{5}$$

If the innovation have a Gaussian distribution, then $\nu_{ij}\nu_{ij}^T$ will form a χ^2 distribution. This is the basis of a validation gate approach: observations inside a fixed region of the defined χ^2 distribution are accepted, and observations that make the NIS to fall outside this region are rejected. This assumption is achieved by comparing NIS with a threshold value γ_n. This value is defined by fixing the region of acceptance of the χ^2 distribution (e.g., in our experiments, the innovation vector is of dimension 2, and the gate γ_2 equal to 4.0, if z_i is truely an observation of landmark x_j the association will be accepted with 90% of probability).

The validation gate defines a region in the observation space centered about the predicted observation $h(\hat{\mathbf{x}}_j)$. Then, an acceptable observation must fall inside this region. Data association ambiguity occurs if either multiple observations fall within the validation gates of a particular target, or a single observation lies within the gates of multiple targets. Furthermore, it is possible that an observation might arise from clutter or non-tracked targets leading to false associations even with the satisfaction of unique gating conditions. The most common ambiguity resolution method is nearest neighbour data association. Given a set of observations, $\mathbf{Z}$, within the validation gate of target $\mathbf{x}$, a normalised distance ND_l can be calculated to each $\mathbf{z}_l \in \mathbf{Z}$

$$ND_l = \nu_l^T \mathbf{S}_l^{-1} \nu_l + log|\mathbf{S}_l| \tag{6}$$

Nearest neighbour data association then chooses the observation that minimizes ND_l.

C. EKF update

If a corner already stored in the map as estimate $(\hat{x}_i,\ \hat{y}_i)$ is observed by the laser sensor with the measurement

$$\mathbf{z} = \begin{bmatrix} x \\ y \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} \sigma_{xx}^2 & \sigma_{xy}^2 \\ \sigma_{xy}^2 & \sigma_{yy}^2 \end{bmatrix} \tag{7}$$

where $(x,\ y)$ are the Cartesian coordinates of the observed corner and $\mathbf{R}$ is the observation covariance, then the observed corner is related to the map by

$$\hat{\mathbf{z}}_i = h_i(\hat{\mathbf{x}}_a) = \begin{bmatrix} (\hat{x}_i - \hat{x}_v)\cos\hat{\phi}_v + (\hat{y}_i - \hat{y}_v)\sin\hat{\phi}_v \\ -(\hat{x}_i - \hat{x}_v)\sin\hat{\phi}_v + (\hat{y}_i - \hat{y}_v)\cos\hat{\phi}_v \end{bmatrix} \tag{8}$$

The Kalman gain $\mathbf{K}_i$ can be obtained as

$$\begin{aligned} \nu_i &= \mathbf{z} - h_i(\hat{\mathbf{x}}_a^-) \\ \mathbf{S}_i &= \nabla h_{\mathbf{x}_a} \mathbf{P}_a^- \nabla h_{\mathbf{x}_a}^T + \mathbf{R} \\ \mathbf{K}_i &= \mathbf{P}_a^- \nabla h_{\mathbf{x}_a}^T \mathbf{S}_i^{-1} \end{aligned} \tag{9}$$

where the Jacobian $\nabla h_{\mathbf{x}_a}$ can be obtained as

$$\nabla h_{\mathbf{x}_a} = \left(\frac{\partial h_i}{\partial \mathbf{x}_a} \right)_{\hat{\mathbf{x}}_a^-} \tag{10}$$

It can be noted that the Jabobian $\nabla h_{\mathbf{x}_a}$ only presents non-zero terms align with the positions of the robot states and the observed feature states in the augmented state vector. The posterior SLAM estimate is determined from

$$\hat{\mathbf{x}}_a^+ = \hat{\mathbf{x}}_a^- + \mathbf{K}_i \nu_i \qquad \mathbf{P}_a^+ = \mathbf{P}_a^- - \mathbf{K}_i \mathbf{S}_i \mathbf{K}_i^T \tag{11}$$

In this work, the measurement covariance matrix $\mathbf{R}$ has been obtained from a set of experiments for different laser settings (angle increment) and detected feature distances. In order to extrapolate the values extracted from experiments and obtain the matrix values for any feature distance or laser settings, a polynomial least square fitting has been used for each detected feature. Therefore, each variance in the measurement covariance matrix is a function of the distance. It permits to compute the covariance values for each detected landmark.

In particular, for one degree of angle increment, the values of the matrix are determined by the following expressions:

$$\begin{aligned} \sigma_{xx}^2 &= -6.1465 \cdot 10^{-11}\ d^2 + 7.7383 \cdot d + 9.98563 \cdot 10^{-6} \\ \sigma_{yy}^2 &= -2.4043 \cdot 10^{-8}\ d^2 + 9.5363 \cdot 10^{-7}\ d + 0.01247 \\ \sigma_{xy}^2 &= -4.8967 \cdot 10^{-10}\ d^2 + 2.887 \cdot 10^{-8}\ d + 2.4706 \cdot 10^{-4} \end{aligned} \tag{12}$$

where d is the distance between the landmark and the robot in centimeters. When the laser precision is increased to 0.5 degree of angle increment, the variances are determined by:

$$\begin{aligned} \sigma_{xx}^2 &= -1.3888 \cdot 10^{-10}\ d^2 + 2.0697 \cdot 10^{-7}\ d + 9.599 \cdot 10^{-6} \\ \sigma_{yy}^2 &= -4.08 \cdot 10^{-8}\ d^2 - 1.0506 \cdot 10^{-6}\ d + 0.03577 \\ \sigma_{xy}^2 &= 1.6691 \cdot 10^{-9}\ d^2 - 2.6607 \cdot 10^{-8}\ d - 0.001461 \end{aligned} \tag{13}$$

The main advantage of using an adaptive covariance matrix is that distant corners are associated to higher variances than closer corners. This provides more precision to closer detected corners. Besides, distant corners can be employed in the updating stage, although their associated covariance matrices provide them less precision. In this way, we avoid to reject distant corners and the corresponding reduction of the set of possible correct associations.

D. State vector augmentation

When the environment is explored, new landmarks are observed and must be added to the map under certain conditions. To initialise new landmarks, the state vector and covariance matrix are augmented with the values of the new observation,

z, and its covariance, $\mathbf{R}$, as a relative measure to the robot. In the described approach, we employ the standard procedure for state vector augmentation. Thus, new landmarks are added to a *potential* list [12], and only if they are observed several times, they are transferred from this list to the confirmed map.

Then, when a new observation is obtained, the described algorithm firstly tries to associate it with a landmark stored in the confirmed map. If this association fails, it tries to associate it with a landmark in the *potential* list. Failing that, it becomes a new *potential* landmark. As it is commented above, several associations to a particular potential landmark will transfer it to the map. Similarly, a potential landmark will be demised when several observations after its inclusion in the list do not associate any observed measurement with it.

In order to increase landmark stability, a quality index is computed for each landmark stored in the map [12]. When a stored landmark is observed again, its quality index is recomputed and landmarks that do not achieve a minimim threshold can be deleted from the map.

III. Experimental Results

Figs. 1.a shows a experimental result obtained by running the proposed algorithm. Figure illustrates the odometry and the estimated trajectory. The robot pose uncertainty has been drawn over the trajectory. The major axis of the final uncertainty ellipse is 1.6 m. Fig. 1b shows the determinant of the robot pose covariance. It can be noted the high rate of robot pose updating due to the dense presence of landmarks. This fact is granted by the existence of distant landmarks, which can be included without disturbing the whole process due to the use of an adaptive observation covariance. Finally, it must be noted that the whole EKF–SLAM algorithm runs every 300 ms on the 400MHz Versak6 PC 104+ embedded on our Pioneer 2AT mobile platform. The landmark acquisition algorithm only takes 25 ms including 180^o laser data acquisition (angle increment of 0.5 degree).

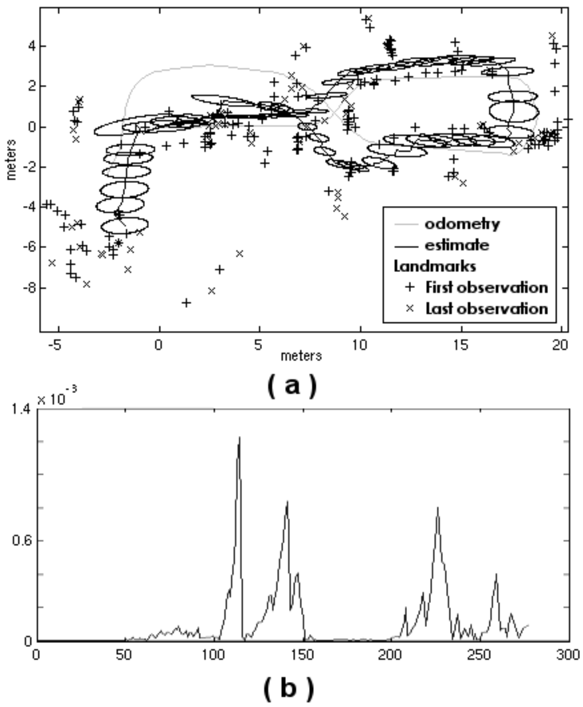

Fig. 1. a) Estimated trajectory of the robot; and b) determinant of the robot pose covariance

IV. Conclusions and Future Work

Experiments show that the adaptive observation covariance permits to introduce more landmarks for the updating stage of the EKF–based SLAM and thus, its efficiency is improved. On the other hand, the proposed landmark extraction algorithm reduces the computational cost associated to the whole process. This fact is specially interesting because it avoids large periods without updating. The increasing of the updating rate reduces the robot pose uncertainty and avoids that data association becomes very fragile [1].

Future work will be focused on reducing the linearization errors inherent to the EKF–SLAM process [1]. The proposed method can be combined with local maps or robocentric mapping [3] to further improve map consistency.

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